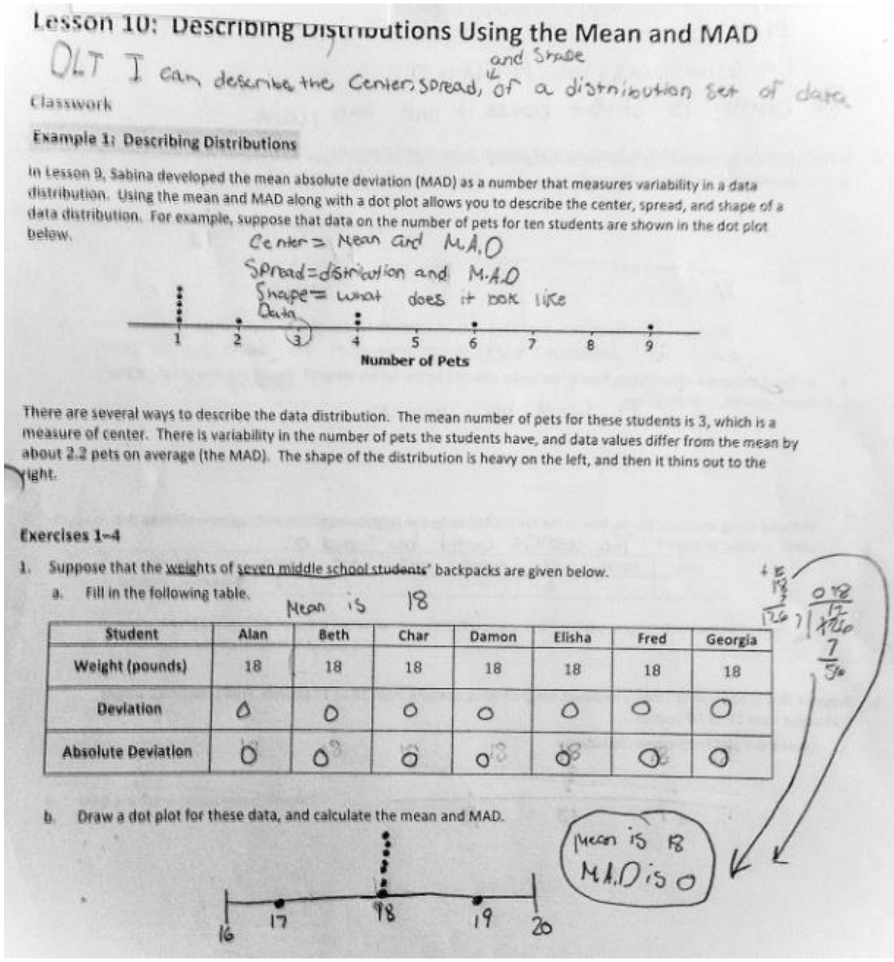
Module 1: Section 1D: A Closer Look at the Standards for Mathematical Content: Sixth Grade Sample Tasks

Task 1:



Lesson 10: Describing Distributions Using the Mean and MAD

OLT I can describe the Center, Spread, and Shape of a distribution set of data

Classwork

Example 1: Describing Distributions

In Lesson 9, Sabina developed the mean absolute deviation (MAD) as a number that measures variability in a data distribution. Using the mean and MAD along with a dot plot allows you to describe the center, spread, and shape of a data distribution. For example, suppose that data on the number of pets for ten students are shown in the dot plot below.

Center = Mean and M.A.D

Spread = distribution and M.A.D

Shape = what does it look like Data

Number of Pets

There are several ways to describe the data distribution. The mean number of pets for these students is 3, which is a measure of center. There is variability in the number of pets the students have, and data values differ from the mean by about 2.2 pets on average (the MAD). The shape of the distribution is heavy on the left, and then it thins out to the right.

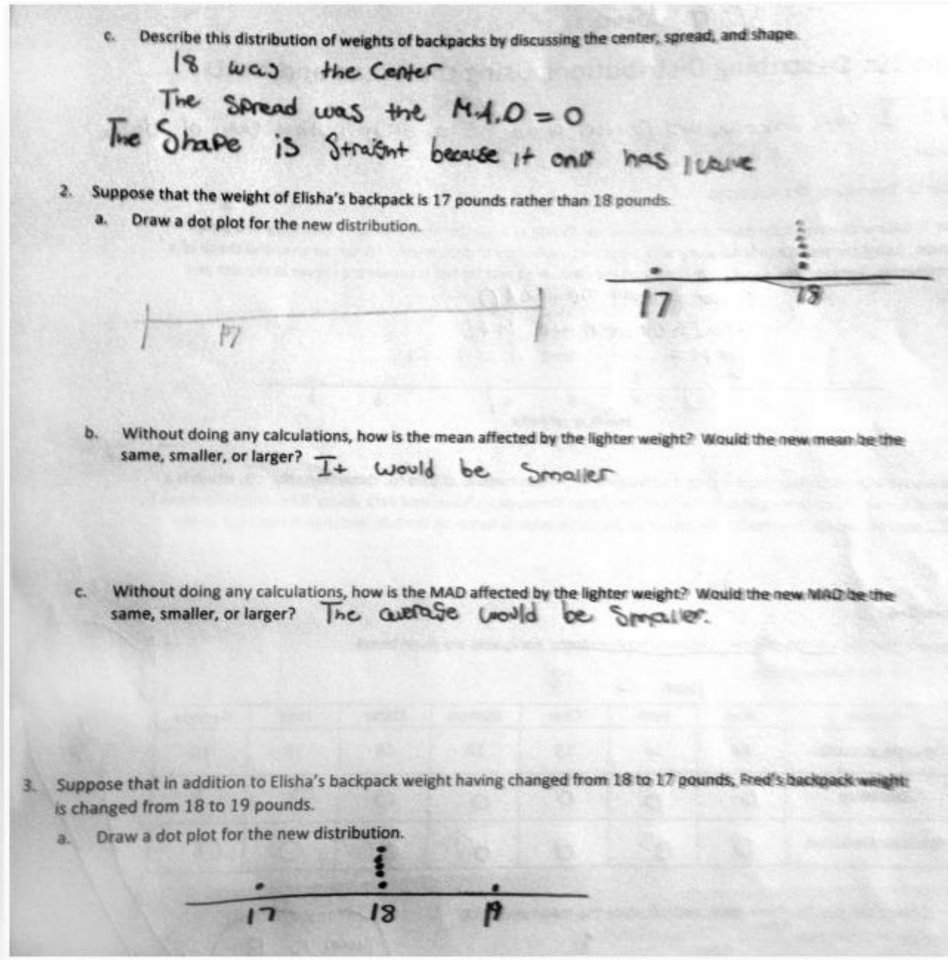
Exercises 1–4

1. Suppose that the weights of seven middle school students' backpacks are given below.
   a. Fill in the following table. Mean is 18

Student	Alan	Beth	Char	Damon	Elisha	Fred	Georgia
Weight (pounds)	18	18	18	18	18	18	18
Deviation	0	0	0	0	0	0	0
Absolute Deviation	0	0	0	0	0	0	0

   b. Draw a dot plot for these data, and calculate the mean and MAD.

Mean is 18
M.A.D is 0



   c. Describe this distribution of weights of backpacks by discussing the center, spread, and shape.

18 was the Center
The Spread was the M.A.D = 0
The Shape is Straight because it only has 1 value

2. Suppose that the weight of Elisha's backpack is 17 pounds rather than 18 pounds.
   a. Draw a dot plot for the new distribution.

   b. Without doing any calculations, how is the mean affected by the lighter weight? Would the new mean be the same, smaller, or larger? It would be Smaller

   c. Without doing any calculations, how is the MAD affected by the lighter weight? Would the new MAD be the same, smaller, or larger? The average would be Smaller.

3. Suppose that in addition to Elisha's backpack weight having changed from 18 to 17 pounds, Fred's backpack weight is changed from 18 to 19 pounds.
   a. Draw a dot plot for the new distribution.

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b. Without doing any calculations, how would the new mean compare to the original mean?

The mean would be the Same

c. Without doing any calculations, would the MAD for the new distribution be the same as, smaller than, or larger than the original MAD? It woold be kerger becqose of the spread.

d. Without doing any calculations, how would the MAD for the new distribution compare to the one in Exercise 2? The mad in this one is bigger because it has a larger spread.

4. Suppose that seven second graders' backpack weights were as follows:

Student	Alice	Bob	Carol	Damon	Ed	Felipe	Gale
Weight (pounds)	5	5	5	5	5	5	5

a. How is the distribution of backpack weights for the second graders similar to the original distribution for the middle school students given in Exercise 1?

b. How are the distributions different?

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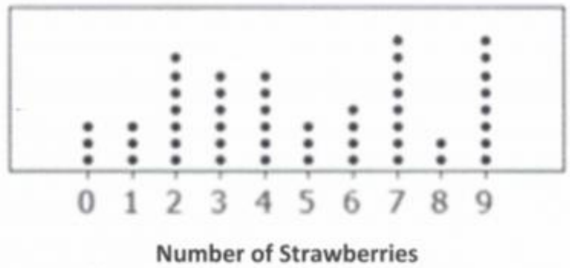
Task 2:



Define and calculate the range.

Range = the difference between the smallest and biggest #

smallest = 0

biggest = 9

$9 - 0 = \boxed{9}$

Define and calculate the median.

49 pieces of data — one number in the middle

0,0,0,1,1,1,2,2,2,2,2,2,2,3,3,3,3,3,3,
4,4,4,4,4,$\boxed{4}$,5,5,5,6,6,6,6,7,7,7,7,7,
7,7,7,8,8,9,9,9,9,9,9,9,

$49 + 1 = 50\ (25)$

$49 - 1 = 48\ (24)$

median = $\boxed{4}$

Define and calculate the mean.

mean = average

add up all of the number in the data and then divide by the number of data pieces you have

$\text{mean} = 3(0) + 3(1) + 7(2) + 6(3) + 6(4) + 3(5) + 4(6) + 8(7) + 2(8) + 7(9)$

$\frac{233}{49} = \boxed{4.76}$

Define and calculate the mean absolute deviation (MAD).

MAD = average of how the difference from the data points to the mean

$\text{MAD} = 3(4.76) + 3(3.76) + 7(2.76) + 6(1.76) + 6(0.76) + 3(0.24) + 4(1.24) + 8(2.24) + 2(3.24) + 7(4.24)$

$14.28 + 25.56 + 19.32 + 10.56 + 4.56 + 0.72 + 4.96 + 17.92 + 6.48 + 29.68$

$\frac{134.04}{49} = \boxed{2.74}$

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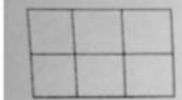
Task 3:

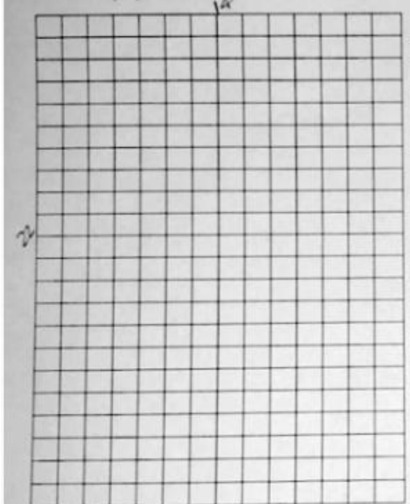
Name ______________________

Area of Square and Rectangles

Think About it



How many square units are there in a rectangle above? 6 unit



How many square units in the rectangle above? 308 units

What is the most efficient way to determine the number of square units in the rectangle above! Why does that way work?

Multiply length and width. You are counting.

Area: 22 × 14

Ex 1) Tyson owns a business that creates rugs. He is working on making a rectangular rug that is seven feet long and three feet wide. What is area of the rug?

A = l · w
A = 3 · 7

Answer: 21

Why do we use square units instead of another type of unit (such as circular units)?

Because 1×1 is 1 and all sides are equal.

Example 2) Tyson dedicated to make a square rug with a side length of 8 feet.

A) How much space is being covered by the rug?

A = LW
A = 8 · 3

Answer: 24

B) If the rug is composed of pieces of fabrics that are 2 feet by 2 feet, how many pieces of fabrics you need?

2×2=4F

Answer: 6

Example 3) Tyson was asked to make a rug that will cover a rectangular floor that has a width of 18 feet and a length of 25 feet. How many square feet of carpeting will he need to cover the entire floor?

A = LW
A = 25 × 18

Answer: 450^2 ft

$$\begin{array}{r} 25 \\ \times\ 18 \\ \hline 200 \\ +250 \\ \hline 450 \end{array}$$

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Participant Guide

<table>
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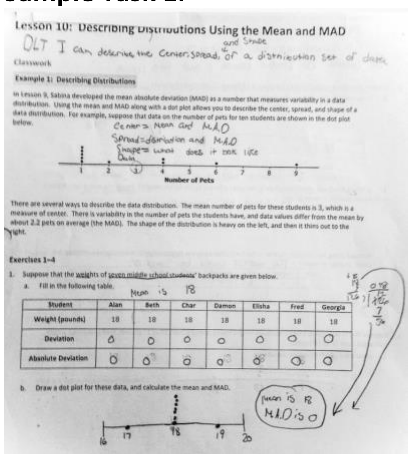
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Shape = what does it look like
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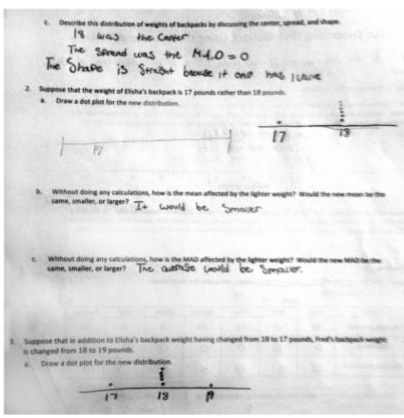
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a. Draw a dot plot for the new distribution.</td><td>Can you identify the targeted content standard(s) for this task?</td><td>None/Weak
Partial
Strong</td><td>Can you identify the targeted practice standard(s) for this task?</td></tr>
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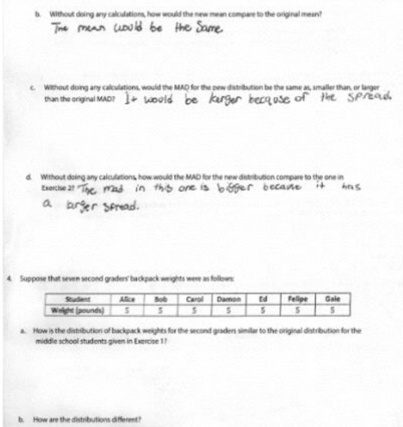
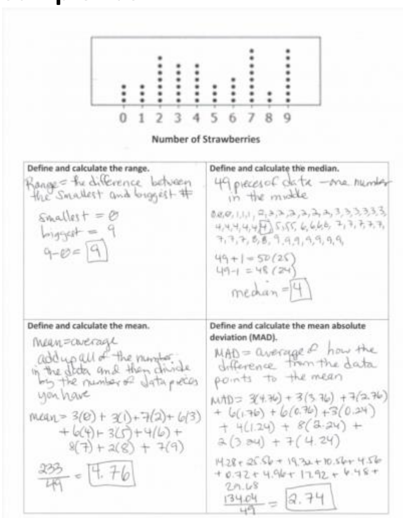
			
Sample Task 2: 	Can you identify the targeted content standard(s) for this task?	• None/Weak • Partial • Strong	Can you identify the targeted practice standard(s) for this task?

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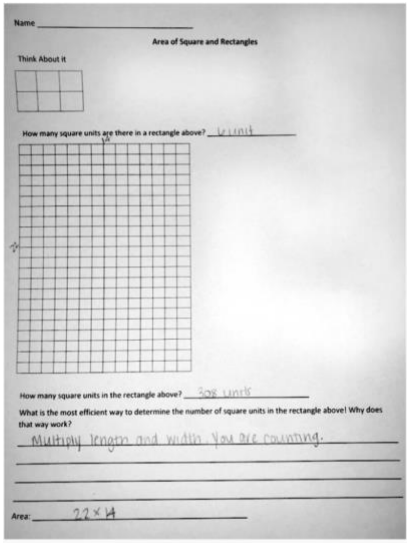
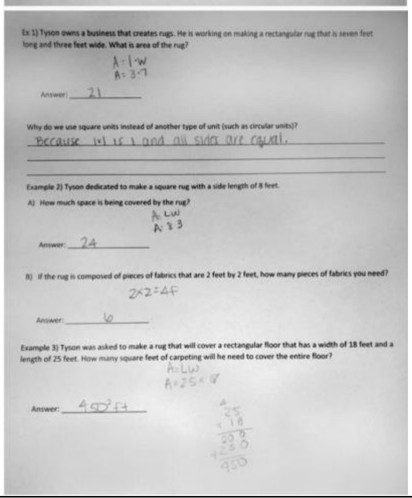

Name ____________
Area of Square and Rectangles
Think About it
How many square units are there in a rectangle above? 6 unit
How many square units in the rectangle above? 308 units
What is the most efficient way to determine the number of square units in the rectangle above! Why does that way work?
Multiply length and width. You are counting.
Area: 22 × 14

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A = LW
A = 25 × 18
Answer: 450² ft
25 × 18 = 200 + 250 = 450</td>
<td>Can you identify the targeted content standard(s) for this task?</td>
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Module 1: Section 1D: A Closer Look at the Standards for Mathematical Content: Sixth Grade Sample Tasks

Facilitator's Guide

Throughout facilitation of this activity it will be important to remind participants:

- Use the grade-level overview to determine the relevant cluster(s) to look at more closely
- Questions regarding Standards for Mathematical Practices will only be indicated where specific practices were identified within the source of the task alignment. Additionally, emphasize to participants the statement at the end of each cluster within the *KAS for Mathematics*, "The identified mathematical practices, coherence connections, and clarifications are possible suggestions; however, they are not the only pathways."

Sample Task 1:

This assignment is strongly aligned to the standards.

Overview

Sixth-grade students draw dot plots to represent data sets, calculate the mean and mean absolute deviation, and explain how the values of the mean and mean absolute deviation would change if at least one value in the data set changed. This assignment is strong because it not only builds students' skill in calculating these measures, but also builds their conceptual understanding of the measures by asking students to describe and explain them.

Related Standards

We looked at how well the assignment aligned to the following standards:

KY.6.SP.4 Display the distribution of numerical data in plots on a number line, including dot plots, histograms and box plots.

KY.6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Determining quantitative measures of center (median and/or mean) to describe distribution of numerical data.
d. Describing distributions of numerical data qualitatively relating to shape (using terms such as cluster, mode(s), gap, symmetric, uniform, skewed-left, skewed-right and the presence of outliers) and quantitatively relating to spread/variability (using terms such as range and interquartile range).
e. Relating the choice of measures of center and variability to the shape of the data distribution.

Why is this assignment strongly aligned?

This assignment aligns with two sixth-grade standards:

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- KY.6.SP.4 requires students to display data graphically in a variety of ways on a number line, and this assignment prompts students to represent three data sets on a dot plot—a format referenced in the standard.
- KY.6.SP.5 (parts b-e) requires students to calculate measures of center (median and mean) and measures of variability (interquartile range), and also describe these measures within the context of the data set. For this assignment, students had to calculate the mean and mean absolute deviation for three data sets. They also had to describe the data distributions (problem 1c), explain the meaning of the values of the mean and mean absolute deviation (problem 3c), and explain how the values of the mean and mean absolute deviation would change if one or more values in the data set changed (problems 2b-c). (interquartile range), and also describe these measures within the context of the data set. For this assignment, students had to calculate the mean and mean absolute deviation for three data sets. They also had to describe the data distributions (problem 1c), explain the meaning of the values of the mean and mean absolute deviation (problem 3c), and explain how the values of the mean and mean absolute deviation would change if one or more values in the data set changed (problems 2b-c).

This assignment focuses on both conceptual understanding and procedural skill, both of which are targeted in standards KY.6.SP.4 and KY.6.SP.5. Drawing dot plots and calculating mean and mean absolute deviation allows students to build procedural skill. Students build their conceptual understanding by providing descriptions and explanations of the measures of center and variability. For example, in the problems that ask students to explain how the value of the mean would change if a value in the data set changed, students are asked to <u>not</u> calculate the value of the new mean. Asking students to explain without doing actual calculations is a good way to get them to articulate their understanding of what the mean represents and how individual data points affect it. Drawing dot plots and calculating mean and mean absolute deviation allows students to build procedural skill. Students build their conceptual understanding by providing descriptions and explanations of the measures of center and variability. For example, in the problems that ask students to explain how the value of the mean would change if a value in the data set changed, students are asked to <u>not</u> calculate the value of the new mean. Asking students to explain without doing actual calculations is a good way to get them to articulate their understanding of what the mean represents and how individual data points affect it.

Practice Standards

This assignment allows students to engage with multiple mathematical practice standards. Students engage with Mathematical Practice Standard #4 ("Model with mathematics") by mathematically representing real-world topics—like backpack weights—with dot plots. They engage with Mathematical Practice Standard #3 ("Construct viable arguments and critique the reasoning of others") and Mathematical Practice Standard #6 ("Attend to precision") by explaining how the values of the mean and mean absolute deviation would change given a new data point and agreeing or disagreeing with another student's reasoning (problem 2).

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Sample Task 2:

This assignment is partially aligned to the standards.

Overview

Sixth-grade students define and calculate range, median, mean, and mean absolute deviation for a provided data set. This assignment is only partially aligned with a sixth-grade standard. Calculating these values is appropriate, but the assignment doesn't ask students to describe patterns and deviations in the data set, as the sixth-grade standard requires.

Related Standards

We looked at how well the assignment aligned to the following standard:

KY.6.SP.5 Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- c. Determining quantitative measures of center (median and/or mean) to describe distribution of numerical data.
- d. Describing distributions of numerical data qualitatively relating to shape (using terms such as cluster, mode(s), gap, symmetric, uniform, skewed-left, skewed-right and the presence of outliers) and quantitatively relating to spread/variability (using terms such as range and interquartile range).
- e. Relating the choice of measures of center and variability to the shape of the data distribution.

Why is this assignment partially aligned?

This assignment is partially aligned with sixth-grade standard KY.6.SP.5. Students are asked to calculate measures of center and measures of variability, which is called for in the standard. To be fully aligned with the standard, however, the assignment should have asked students to describe patterns or deviations in the data. For example, students could have been asked what the values of the mean and mean absolute deviation tell us about the number of strawberries in the data set.

This assignment only focuses on procedural skill, but standard KY.6.SP.5 targets both procedural skill and conceptual understanding. Students build their procedural skill in this assignment by calculating range, mean, median, and mean absolute deviation. But they don't get to build their conceptual understanding because they aren't asked to interpret and describe these measures of center and variability in the context of the specific data set. For example, students could have been asked to describe the distribution as symmetric or non-symmetric, and explain how that relates to the values of the mean and median, to reinforce their conceptual understanding of variability (the closer in value the mean and median, the more even or symmetric the distribution will be).

Practice Standards

Students had a superficial opportunity to engage with Mathematical Practice Standard #6 ("Attend to precision") through their definitions of range, mean, median, and mean absolute deviation, but they didn't get a meaningful opportunity to communicate precisely because they were not asked to describe the measures of center and variability within the data set.

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Sample Task 3:

This assignment is weakly aligned to the standards.

Overview

Sixth-grade students answer several questions about calculating area. This assignment is weak because it is more closely aligned with a fourth-grade standard on calculating the area of rectangles than with the sixth-grade standard on finding the area of more advanced shapes.

Related Standards

We looked at how well the assignment aligned to the following standard:

KY.6.G.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Why is this assignment weakly aligned?

The sixth-grade standard KY.6.G.1 requires students to find the area of a variety of shapes, such as triangles, “special” quadrilaterals (like parallelograms), and other polygons (like pentagons). The only shapes in this assignment are rectangles, making this assignment more closely aligned with fourth-grade standard KY.4.MD.3. There was also a missed opportunity to incorporate multiplication appropriate for sixth grade. The multiplication students perform in this assignment (e.g., 3 x 7 and 14 x 22) is below grade level; students should have been asked to work with non-whole numbers.

The assignment doesn’t focus on conceptual understanding, which is an essential aspect of standard KY.6.G.1. The standard calls for students to find the area of a shape “by composing [it] into rectangles or decomposing [it] into triangles and other shapes.” Composing and decomposing shapes builds students’ foundational understanding of the area formula for various shapes. For example, seeing that a parallelogram can be composed of two triangles helps students understand that area is additive and provides an explanation for why the formula for area of a triangle is (½ x base x height). In this assignment, students are only asked to calculate area using the (base x height) formula, which reinforces procedural skill, not conceptual understanding.

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